Mathematica 11.3 Integration Test Results

Test results for the 100 problems in "7.5.2 Inverse hyperbolic secant functions.m"

Problem 1: Result unnecessarily involves imaginary or complex numbers.

$$\int x^3 \operatorname{ArcSech}[a+b x] \, dx$$

Optimal (type 3, 203 leaves, 8 steps):

$$-\frac{\left(2+17 a^2\right) \sqrt{\frac{1-a-b x}{1+a+b x}} (1+a+b x)}{12 b^4}-\frac{x^2 \sqrt{\frac{1-a-b x}{1+a+b x}} (1+a+b x)}{12 b^2}+\frac{a (a+b x) \sqrt{\frac{1-a-b x}{1+a+b x}} (1+a+b x)}{3 b^4}-\frac{a^4 \operatorname{ArcSech}[a+b x]}{4 b^4}+\frac{1}{4} x^4 \operatorname{ArcSech}[a+b x]+\frac{a \left(1+2 a^2\right) \operatorname{ArcTan}\left[\frac{\sqrt{\frac{1-a-b x}{1+a+b x}} (1+a+b x)}{a+b x}\right]}{2 b^4}$$

Result (type 3, 225 leaves):

$$-\frac{1}{12 b^4}\left(\sqrt{-\frac{-1+a+b x}{1+a+b x}} \left(2+2 a+13 a^2+13 a^3+\left(2-4 a+9 a^2\right) b x+(1-3 a) b^2 x^2+b^3 x^3\right)-3 b^4 x^4 \operatorname{ArcSech}[a+b x]-3 a^4 \operatorname{Log}[a+b x]+3 a^4 \operatorname{Log}\left[1+\sqrt{-\frac{-1+a+b x}{1+a+b x}}+a \sqrt{-\frac{-1+a+b x}{1+a+b x}}+b x \sqrt{-\frac{-1+a+b x}{1+a+b x}}\right]+6 i a \left(1+2 a^2\right) \operatorname{Log}\left[-2 i (a+b x)+2 \sqrt{-\frac{-1+a+b x}{1+a+b x}} (1+a+b x)\right]\right)$$

Problem 2: Result unnecessarily involves imaginary or complex numbers.

$$\int x^2 \operatorname{ArcSech}[a+b x] \, dx$$

Optimal (type 3, 153 leaves, 7 steps):

$$\frac{5\,a\,\sqrt{\frac{1-a-b\,x}{1+a+b\,x}}\,(1+a+b\,x)}{6\,b^3}-\frac{x\,\sqrt{\frac{1-a-b\,x}{1+a+b\,x}}\,(1+a+b\,x)}{6\,b^2}+\frac{a^3\,\mathrm{ArcSech}[a+b\,x]}{3\,b^3}+\frac{1}{3}\,x^3\,\mathrm{ArcSech}[a+b\,x]-\frac{(1+6\,a^2)\,\mathrm{ArcTan}\left[\frac{\sqrt{\frac{1-a-b\,x}{1+a+b\,x}}\,(1+a+b\,x)}{a+b\,x}\right]}{6\,b^3}$$

Result (type 3, 200 leaves):

$$\frac{1}{6\,b^3}\left(\sqrt{-\frac{-1+a+b\,x}{1+a+b\,x}}\,\left(5\,a^2-b\,x\,(1+b\,x)+a\,(5+4\,b\,x)\right)+2\,b^3\,x^3\,\mathrm{ArcSech}[a+b\,x]-2\,a^3\,\mathrm{Log}[a+b\,x]+2\,a^3\,\mathrm{Log}\left[1+\sqrt{-\frac{-1+a+b\,x}{1+a+b\,x}}+a\,\sqrt{-\frac{-1+a+b\,x}{1+a+b\,x}}+b\,x\,\sqrt{-\frac{-1+a+b\,x}{1+a+b\,x}}\right]+i\,(1+6\,a^2)\,\mathrm{Log}\left[-2\,i\,(a+b\,x)+2\,\sqrt{-\frac{-1+a+b\,x}{1+a+b\,x}}\,(1+a+b\,x)\right]\right)$$

Problem 3: Result unnecessarily involves imaginary or complex numbers.

$$\int x\,\mathrm{ArcSech}[a+b\,x]\,dx$$

Optimal (type 3, 107 leaves, 6 steps):

$$-\frac{\sqrt{\frac{1-a-b\,x}{1+a+b\,x}}\,(1+a+b\,x)}{2\,b^2}-\frac{a^2\,\mathrm{ArcSech}[a+b\,x]}{2\,b^2}+\frac{1}{2}\,x^2\,\mathrm{ArcSech}[a+b\,x]+\frac{a\,\mathrm{ArcTan}\left[\frac{\sqrt{\frac{1-a-b\,x}{1+a+b\,x}}\,(1+a+b\,x)}{a+b\,x}\right]}{b^2}$$

Result (type 3, 176 leaves):

$$\frac{1}{2\,b^2}\left(-\sqrt{-\frac{-1+a+b\,x}{1+a+b\,x}}\,(1+a+b\,x)+b^2\,x^2\,\mathrm{ArcSech}[a+b\,x]+a^2\,\mathrm{Log}[a+b\,x]-a^2\,\mathrm{Log}\left[1+\sqrt{-\frac{-1+a+b\,x}{1+a+b\,x}}+a\,\sqrt{-\frac{-1+a+b\,x}{1+a+b\,x}}+b\,x\,\sqrt{-\frac{-1+a+b\,x}{1+a+b\,x}}\right]-2\,i\,a\,\mathrm{Log}\left[-2\,i\,(a+b\,x)+2\,\sqrt{-\frac{-1+a+b\,x}{1+a+b\,x}}\,(1+a+b\,x)\right]\right)$$

Problem 4: Result more than twice size of optimal antiderivative.

$$\int \mathrm{ArcSech}[a+b\,x]\,dx$$

Optimal (type 3, 44 leaves, 4 steps):

$$\frac{(a+b\,x)\,\text{ArcSech}[a+b\,x]}{b}-\frac{2\,\text{ArcTan}\left[\sqrt{\frac{1-a-b\,x}{1+a+b\,x}}\right]}{b}$$

Result (type 3, 105 leaves):

$$x\,\text{ArcSech}[a+b\,x]-\frac{1}{b\sqrt{\frac{-1+a+b\,x}{1+a+b\,x}}}$$
$$\sqrt{-\frac{-1+a+b\,x}{1+a+b\,x}}\left(a\,\text{ArcTan}\left[\frac{1}{\sqrt{-1+a+b\,x}\,\sqrt{1+a+b\,x}}\right]+\text{Log}\left[a+b\,x+\sqrt{-1+a+b\,x}\,\sqrt{1+a+b\,x}\right]\right)$$

Problem 5: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{\text{ArcSech}[a+b\,x]}{x}\,dx$$

Optimal (type 4, 170 leaves, 14 steps):

$$\text{ArcSech}[a+b\,x]\,\text{Log}\left[1-\frac{a\,e^{\text{ArcSech}[a+b\,x]}}{1-\sqrt{1-a^2}}\right]+\text{ArcSech}[a+b\,x]\,\text{Log}\left[1-\frac{a\,e^{\text{ArcSech}[a+b\,x]}}{1+\sqrt{1-a^2}}\right]-$$
$$\text{ArcSech}[a+b\,x]\,\text{Log}\left[1+e^{2\,\text{ArcSech}[a+b\,x]}\right]+\text{PolyLog}\left[2,\frac{a\,e^{\text{ArcSech}[a+b\,x]}}{1-\sqrt{1-a^2}}\right]+$$
$$\text{PolyLog}\left[2,\frac{a\,e^{\text{ArcSech}[a+b\,x]}}{1+\sqrt{1-a^2}}\right]-\frac{1}{2}\,\text{PolyLog}\left[2,-e^{2\,\text{ArcSech}[a+b\,x]}\right]$$

Result (type 4, 332 leaves):

$$-4\,i\,\text{ArcSin}\left[\frac{\sqrt{\frac{-1+a}{a}}}{\sqrt{2}}\right]\,\text{ArcTanh}\left[\frac{(1+a)\,\text{Tanh}\left[\frac{1}{2}\text{ArcSech}[a+b\,x]\right]}{\sqrt{1-a^2}}\right]-$$

$$\text{ArcSech}[a+b\,x]\,\text{Log}\left[1+e^{-2\,\text{ArcSech}[a+b\,x]}\right]+\text{ArcSech}[a+b\,x]\,\text{Log}\left[1+\frac{\left(-1+\sqrt{1-a^2}\right)\,e^{-\text{ArcSech}[a+b\,x]}}{a}\right]+$$

$$2\,i\,\text{ArcSin}\left[\frac{\sqrt{\frac{-1+a}{a}}}{\sqrt{2}}\right]\,\text{Log}\left[1+\frac{\left(-1+\sqrt{1-a^2}\right)\,e^{-\text{ArcSech}[a+b\,x]}}{a}\right]+$$

$$\text{ArcSech}[a+b\,x]\,\text{Log}\left[1-\frac{\left(1+\sqrt{1-a^2}\right)\,e^{-\text{ArcSech}[a+b\,x]}}{a}\right]-$$

$$2\,i\,\text{ArcSin}\left[\frac{\sqrt{\frac{-1+a}{a}}}{\sqrt{2}}\right]\,\text{Log}\left[1-\frac{\left(1+\sqrt{1-a^2}\right)\,e^{-\text{ArcSech}[a+b\,x]}}{a}\right]+\frac{1}{2}\,\text{PolyLog}\left[2,-e^{-2\,\text{ArcSech}[a+b\,x]}\right]-$$

$$\text{PolyLog}\left[2,-\frac{\left(-1+\sqrt{1-a^2}\right)\,e^{-\text{ArcSech}[a+b\,x]}}{a}\right]-\text{PolyLog}\left[2,\frac{\left(1+\sqrt{1-a^2}\right)\,e^{-\text{ArcSech}[a+b\,x]}}{a}\right]$$

Problem 6: Result more than twice size of optimal antiderivative.

$$\int\frac{\text{ArcSech}[a+b\,x]}{x^2}\,dx$$

Optimal (type 3, 70 leaves, 5 steps):

$$-\frac{b\,\text{ArcSech}[a+b\,x]}{a}-\frac{\text{ArcSech}[a+b\,x]}{x}+\frac{2\,b\,\text{ArcTanh}\left[\frac{\sqrt{1+a}\,\text{Tanh}\left[\frac{1}{2}\text{ArcSech}[a+b\,x]\right]}{\sqrt{1-a}}\right]}{a\,\sqrt{1-a^2}}$$

Result (type 3, 244 leaves):

$$-\frac{\text{ArcSech}[a+b\,x]}{x}+\frac{1}{a\,\sqrt{1-a^2}}b\left(-\text{Log}[x]+\sqrt{1-a^2}\,\text{Log}[a+b\,x]-\right.$$

$$\sqrt{1-a^2}\,\text{Log}\left[1+\sqrt{-\frac{-1+a+b\,x}{1+a+b\,x}}+a\,\sqrt{-\frac{-1+a+b\,x}{1+a+b\,x}}+b\,x\,\sqrt{-\frac{-1+a+b\,x}{1+a+b\,x}}\right]+\text{Log}\left[1-a^2-a\,b\,x+\right.$$

$$\left.\left.\sqrt{1-a^2}\,\sqrt{-\frac{-1+a+b\,x}{1+a+b\,x}}+a\,\sqrt{1-a^2}\,\sqrt{-\frac{-1+a+b\,x}{1+a+b\,x}}+\sqrt{1-a^2}\,b\,x\,\sqrt{-\frac{-1+a+b\,x}{1+a+b\,x}}\right]\right)$$

Problem 7: Result more than twice size of optimal antiderivative.

$$\int\frac{\text{ArcSech}[a+b\,x]}{x^3}\,dx$$

Optimal (type 3, 133 leaves, 7 steps):

$$\frac{b\sqrt{\frac{1-a-b\,x}{1+a+b\,x}}\,(1+a+b\,x)}{2\,a\,(1-a^2)\,x}+\frac{b^2\,\text{ArcSech}[a+b\,x]}{2\,a^2}-\frac{\text{ArcSech}[a+b\,x]}{2\,x^2}-\frac{(1-2\,a^2)\,b^2\,\text{ArcTanh}\left[\frac{\sqrt{1+a}\,\text{Tanh}\left[\frac{1}{2}\text{ArcSech}[a+b\,x]\right]}{\sqrt{1-a}}\right]}{a^2\,(1-a^2)^{3/2}}$$

Result (type 3, 315 leaves):

$$\frac{1}{2}\left(-\frac{b\sqrt{-\frac{-1+a+b\,x}{1+a+b\,x}}\,(1+a+b\,x)}{(-1+a)\,a\,(1+a)\,x}-\frac{\text{ArcSech}[a+b\,x]}{x^2}-\frac{(-1+2\,a^2)\,b^2\,\text{Log}[x]}{a^2\,(1-a^2)^{3/2}}-\frac{b^2\,\text{Log}[a+b\,x]}{a^2}+\frac{b^2\,\text{Log}\left[1+\sqrt{-\frac{-1+a+b\,x}{1+a+b\,x}}+a\sqrt{-\frac{-1+a+b\,x}{1+a+b\,x}}+b\,x\sqrt{-\frac{-1+a+b\,x}{1+a+b\,x}}\right]}{a^2}+\frac{1}{a^2\,(1-a^2)^{3/2}}(-1+2\,a^2)\,b^2\,\text{Log}\left[1-a^2-a\,b\,x+\sqrt{1-a^2}\sqrt{-\frac{-1+a+b\,x}{1+a+b\,x}}+a\sqrt{1-a^2}\sqrt{-\frac{-1+a+b\,x}{1+a+b\,x}}+\sqrt{1-a^2}\,b\,x\sqrt{-\frac{-1+a+b\,x}{1+a+b\,x}}\right]\right)$$

Problem 12: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int\frac{\text{ArcSech}[a+b\,x]^2}{x}\,dx$$

Optimal (type 4, 274 leaves, 17 steps):

$$\text{ArcSech}[a+b\,x]^2\,\text{Log}\left[1-\frac{a\,e^{\text{ArcSech}[a+b\,x]}}{1-\sqrt{1-a^2}}\right]+\text{ArcSech}[a+b\,x]^2\,\text{Log}\left[1-\frac{a\,e^{\text{ArcSech}[a+b\,x]}}{1+\sqrt{1-a^2}}\right]-\text{ArcSech}[a+b\,x]^2\,\text{Log}\left[1+e^{2\,\text{ArcSech}[a+b\,x]}\right]+2\,\text{ArcSech}[a+b\,x]\,\text{PolyLog}\left[2,\frac{a\,e^{\text{ArcSech}[a+b\,x]}}{1-\sqrt{1-a^2}}\right]+2\,\text{ArcSech}[a+b\,x]\,\text{PolyLog}\left[2,\frac{a\,e^{\text{ArcSech}[a+b\,x]}}{1+\sqrt{1-a^2}}\right]-\text{ArcSech}[a+b\,x]\,\text{PolyLog}\left[2,-e^{2\,\text{ArcSech}[a+b\,x]}\right]-2\,\text{PolyLog}\left[3,\frac{a\,e^{\text{ArcSech}[a+b\,x]}}{1-\sqrt{1-a^2}}\right]-2\,\text{PolyLog}\left[3,\frac{a\,e^{\text{ArcSech}[a+b\,x]}}{1+\sqrt{1-a^2}}\right]+\frac{1}{2}\text{PolyLog}\left[3,-e^{2\,\text{ArcSech}[a+b\,x]}\right]$$

Result (type 4, 778 leaves):

$$-\frac{2}{3}\text{ArcSech}[a+b x]^3-\text{ArcSech}[a+b x]^2 \text{Log}\left[1+e^{-2 \text{ArcSech}[a+b x]}\right]+$$

$$\text{ArcSech}[a+b x]^2 \text{Log}\left[1+\frac{\left(-1+\sqrt{1-a^2}\right) e^{-\text{ArcSech}[a+b x]}}{a}\right]+$$

$$4 i \text{ArcSech}[a+b x] \text{ArcSin}\left[\frac{\sqrt{\frac{-1+a}{a}}}{\sqrt{2}}\right] \text{Log}\left[1+\frac{\left(-1+\sqrt{1-a^2}\right) e^{-\text{ArcSech}[a+b x]}}{a}\right]+$$

$$\text{ArcSech}[a+b x]^2 \text{Log}\left[1-\frac{\left(1+\sqrt{1-a^2}\right) e^{-\text{ArcSech}[a+b x]}}{a}\right]-$$

$$4 i \text{ArcSech}[a+b x] \text{ArcSin}\left[\frac{\sqrt{\frac{-1+a}{a}}}{\sqrt{2}}\right] \text{Log}\left[1-\frac{\left(1+\sqrt{1-a^2}\right) e^{-\text{ArcSech}[a+b x]}}{a}\right]+$$

$$\text{ArcSech}[a+b x]^2 \text{Log}\left[1+\frac{a e^{\text{ArcSech}[a+b x]}}{-1+\sqrt{1-a^2}}\right]+\text{ArcSech}[a+b x]^2 \text{Log}\left[1-\frac{a e^{\text{ArcSech}[a+b x]}}{1+\sqrt{1-a^2}}\right]-$$

$$\text{ArcSech}[a+b x]^2 \text{Log}\left[1+\frac{\left(-1+\sqrt{1-a^2}\right)\left(1-\sqrt{-\frac{-1+a+b x}{1+a+b x}}\,(1+a+b x)\right)}{a (a+b x)}\right]-$$

$$4 i \text{ArcSech}[a+b x] \text{ArcSin}\left[\frac{\sqrt{\frac{-1+a}{a}}}{\sqrt{2}}\right] \text{Log}\left[1+\frac{\left(-1+\sqrt{1-a^2}\right)\left(1-\sqrt{-\frac{-1+a+b x}{1+a+b x}}\,(1+a+b x)\right)}{a (a+b x)}\right]-$$

$$\text{ArcSech}[a+b x]^2 \text{Log}\left[1+\frac{\left(1+\sqrt{1-a^2}\right)\left(-1+\sqrt{-\frac{-1+a+b x}{1+a+b x}}\,(1+a+b x)\right)}{a (a+b x)}\right]+$$

$$4 i \text{ArcSech}[a+b x] \text{ArcSin}\left[\frac{\sqrt{\frac{-1+a}{a}}}{\sqrt{2}}\right] \text{Log}\left[1+\frac{\left(1+\sqrt{1-a^2}\right)\left(-1+\sqrt{-\frac{-1+a+b x}{1+a+b x}}\,(1+a+b x)\right)}{a (a+b x)}\right]+$$

$$\text{ArcSech}[a+b x] \text{PolyLog}\left[2,-e^{-2 \text{ArcSech}[a+b x]}\right]+2 \text{ArcSech}[a+b x] \text{PolyLog}\left[2,-\frac{a e^{\text{ArcSech}[a+b x]}}{-1+\sqrt{1-a^2}}\right]+$$

$$2 \text{ArcSech}[a+b x] \text{PolyLog}\left[2,\frac{a e^{\text{ArcSech}[a+b x]}}{1+\sqrt{1-a^2}}\right]+\frac{1}{2} \text{PolyLog}\left[3,-e^{-2 \text{ArcSech}[a+b x]}\right]-$$

$$2 \text{PolyLog}\left[3,-\frac{a e^{\text{ArcSech}[a+b x]}}{-1+\sqrt{1-a^2}}\right]-2 \text{PolyLog}\left[3,\frac{a e^{\text{ArcSech}[a+b x]}}{1+\sqrt{1-a^2}}\right]$$

Problem 13: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{\text{ArcSech}[a+b x]^2}{x^2} \, dx$$

Optimal (type 4, 224 leaves, 12 steps):

$$-\frac{b\,\text{ArcSech}[a+b x]^2}{a}-\frac{\text{ArcSech}[a+b x]^2}{x}+\frac{2\,b\,\text{ArcSech}[a+b x]\,\text{Log}\left[1-\frac{a\,e^{\text{ArcSech}[a+b x]}}{1-\sqrt{1-a^2}}\right]}{a\sqrt{1-a^2}}-\frac{2\,b\,\text{ArcSech}[a+b x]\,\text{Log}\left[1-\frac{a\,e^{\text{ArcSech}[a+b x]}}{1+\sqrt{1-a^2}}\right]}{a\sqrt{1-a^2}}+\frac{2\,b\,\text{PolyLog}\left[2,\frac{a\,e^{\text{ArcSech}[a+b x]}}{1-\sqrt{1-a^2}}\right]}{a\sqrt{1-a^2}}-\frac{2\,b\,\text{PolyLog}\left[2,\frac{a\,e^{\text{ArcSech}[a+b x]}}{1+\sqrt{1-a^2}}\right]}{a\sqrt{1-a^2}}$$

Result (type 4, 678 leaves):

$$\frac{1}{a}\left(-\frac{(a+b x)\,\text{ArcSech}[a+b x]^2}{x}+\right.$$

$$\frac{1}{\sqrt{-1+a^2}}\,2\,b\left(2\,\text{ArcSech}[a+b x]\,\text{ArcTan}\left[\frac{(-1+a)\,\text{Coth}\left[\frac{1}{2}\text{ArcSech}[a+b x]\right]}{\sqrt{-1+a^2}}\right]-\right.$$

$$2\,i\,\text{ArcCos}\left[\frac{1}{a}\right]\text{ArcTan}\left[\frac{(1+a)\,\text{Tanh}\left[\frac{1}{2}\text{ArcSech}[a+b x]\right]}{\sqrt{-1+a^2}}\right]+$$

$$\left(\text{ArcCos}\left[\frac{1}{a}\right]+2\left(\text{ArcTan}\left[\frac{(-1+a)\,\text{Coth}\left[\frac{1}{2}\text{ArcSech}[a+b x]\right]}{\sqrt{-1+a^2}}\right]+\right.\right.$$

$$\left.\left.\text{ArcTan}\left[\frac{(1+a)\,\text{Tanh}\left[\frac{1}{2}\text{ArcSech}[a+b x]\right]}{\sqrt{-1+a^2}}\right]\right)\right)\text{Log}\left[\frac{\sqrt{-1+a^2}\;e^{-\frac{1}{2}\text{ArcSech}[a+b x]}}{\sqrt{2}\,\sqrt{a}\,\sqrt{-\frac{b x}{a+b x}}}\right]+$$

$$\left(\text{ArcCos}\left[\frac{1}{a}\right]-2\left(\text{ArcTan}\left[\frac{(-1+a)\,\text{Coth}\left[\frac{1}{2}\text{ArcSech}[a+b x]\right]}{\sqrt{-1+a^2}}\right]+\right.\right.$$

$$\left.\left.\text{ArcTan}\left[\frac{(1+a)\,\text{Tanh}\left[\frac{1}{2}\text{ArcSech}[a+b x]\right]}{\sqrt{-1+a^2}}\right]\right)\right)\text{Log}\left[\frac{\sqrt{-1+a^2}\;e^{\frac{1}{2}\text{ArcSech}[a+b x]}}{\sqrt{2}\,\sqrt{a}\,\sqrt{-\frac{b x}{a+b x}}}\right]-$$

$$\left(\text{ArcCos}\left[\frac{1}{a}\right]+2\,\text{ArcTan}\left[\frac{(1+a)\,\text{Tanh}\left[\frac{1}{2}\text{ArcSech}[a+b\,x]\right]}{\sqrt{-1+a^2}}\right]\right)$$
$$\text{Log}\left[-\left(\left((-1+a)\left(1+a-i\sqrt{-1+a^2}\right)\left(-1+\text{Tanh}\left[\frac{1}{2}\text{ArcSech}[a+b\,x]\right]\right)\right)\Big/\left(a\left(-1+a+i\sqrt{-1+a^2}\,\text{Tanh}\left[\frac{1}{2}\text{ArcSech}[a+b\,x]\right]\right)\right)\right)\right]-$$
$$\left(\text{ArcCos}\left[\frac{1}{a}\right]-2\,\text{ArcTan}\left[\frac{(1+a)\,\text{Tanh}\left[\frac{1}{2}\text{ArcSech}[a+b\,x]\right]}{\sqrt{-1+a^2}}\right]\right)$$
$$\text{Log}\left[\left((-1+a)\left(1+a+i\sqrt{-1+a^2}\right)\left(1+\text{Tanh}\left[\frac{1}{2}\text{ArcSech}[a+b\,x]\right]\right)\right)\Big/\left(a\left(-1+a+i\sqrt{-1+a^2}\,\text{Tanh}\left[\frac{1}{2}\text{ArcSech}[a+b\,x]\right]\right)\right)\right]+$$
$$i\left(\text{PolyLog}\left[2,\left(\left(-1-i\sqrt{-1+a^2}\right)\left(-1+a-i\sqrt{-1+a^2}\,\text{Tanh}\left[\frac{1}{2}\text{ArcSech}[a+b\,x]\right]\right)\right)\Big/\left(a\left(-1+a+i\sqrt{-1+a^2}\,\text{Tanh}\left[\frac{1}{2}\text{ArcSech}[a+b\,x]\right]\right)\right)\right]-\right.$$
$$\left.\left.\text{PolyLog}\left[2,\left(\left(i+\sqrt{-1+a^2}\right)\left(-1+a-i\sqrt{-1+a^2}\,\text{Tanh}\left[\frac{1}{2}\text{ArcSech}[a+b\,x]\right]\right)\right)\Big/\left(a\left(-i(-1+a)+\sqrt{-1+a^2}\,\text{Tanh}\left[\frac{1}{2}\text{ArcSech}[a+b\,x]\right]\right)\right)\right]\right)\right)$$

Problem 14: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int\frac{\text{ArcSech}[a+b\,x]^2}{x^3}\,dx$$

Optimal (type 4, 537 leaves, 23 steps):

$$\frac{b^2 \sqrt{\frac{1-a-b x}{1+a+b x}} \left(1+a+b x\right) \text{ArcSech}[a+b x]}{a \left(1-a^2\right) \left(a+b x\right) \left(1-\frac{a}{a+b x}\right)}+\frac{b^2 \text{ArcSech}[a+b x]^2}{2 a^2}-$$

$$\frac{\text{ArcSech}[a+b x]^2}{2 x^2}+\frac{b^2 \text{ArcSech}[a+b x] \text{Log}\left[1-\frac{a e^{\text{ArcSech}[a+b x]}}{1-\sqrt{1-a^2}}\right]}{a^2 \left(1-a^2\right)^{3/2}}-$$

$$\frac{2 b^2 \text{ArcSech}[a+b x] \text{Log}\left[1-\frac{a e^{\text{ArcSech}[a+b x]}}{1-\sqrt{1-a^2}}\right]}{a^2 \sqrt{1-a^2}}-\frac{b^2 \text{ArcSech}[a+b x] \text{Log}\left[1-\frac{a e^{\text{ArcSech}[a+b x]}}{1+\sqrt{1-a^2}}\right]}{a^2 \left(1-a^2\right)^{3/2}}+$$

$$\frac{2 b^2 \text{ArcSech}[a+b x] \text{Log}\left[1-\frac{a e^{\text{ArcSech}[a+b x]}}{1+\sqrt{1-a^2}}\right]}{a^2 \sqrt{1-a^2}}+\frac{b^2 \text{Log}\left[\frac{x}{a+b x}\right]}{a^2 \left(1-a^2\right)}+\frac{b^2 \text{PolyLog}\left[2,\frac{a e^{\text{ArcSech}[a+b x]}}{1-\sqrt{1-a^2}}\right]}{a^2 \left(1-a^2\right)^{3/2}}-$$

$$\frac{2 b^2 \text{PolyLog}\left[2,\frac{a e^{\text{ArcSech}[a+b x]}}{1-\sqrt{1-a^2}}\right]}{a^2 \sqrt{1-a^2}}-\frac{b^2 \text{PolyLog}\left[2,\frac{a e^{\text{ArcSech}[a+b x]}}{1+\sqrt{1-a^2}}\right]}{a^2 \left(1-a^2\right)^{3/2}}+\frac{2 b^2 \text{PolyLog}\left[2,\frac{a e^{\text{ArcSech}[a+b x]}}{1+\sqrt{1-a^2}}\right]}{a^2 \sqrt{1-a^2}}$$

Result (type 4, 1439 leaves):

$$-\frac{\left(a+b x\right)^2 \text{ArcSech}[a+b x]^2}{2 a^2 x^2}+$$

$$\left(b \text{ArcSech}[a+b x] \left(-a \sqrt{-\frac{-1+a+b x}{1+a+b x}} \left(1+a+b x\right)+\left(-1+a^2\right) \left(a+b x\right) \text{ArcSech}[a+b x]\right)\right)\Big/$$

$$\left(\left(-1+a\right) a^2 \left(1+a\right) x\right)+\frac{b^2 \text{Log}\left[\frac{b x}{a+b x}\right]}{a^2-a^4}-$$

$$\frac{1}{\left(-1+a^2\right)^{3/2}} 2 b^2 \left(2 \text{ArcSech}[a+b x] \text{ArcTan}\left[\frac{\left(-1+a\right) \text{Coth}\left[\frac{1}{2} \text{ArcSech}[a+b x]\right]}{\sqrt{-1+a^2}}\right]-\right.$$

$$2 i \text{ArcCos}\left[\frac{1}{a}\right] \text{ArcTan}\left[\frac{\left(1+a\right) \text{Tanh}\left[\frac{1}{2} \text{ArcSech}[a+b x]\right]}{\sqrt{-1+a^2}}\right]+$$

$$\left(\text{ArcCos}\left[\frac{1}{a}\right]+2 \left(\text{ArcTan}\left[\frac{\left(-1+a\right) \text{Coth}\left[\frac{1}{2} \text{ArcSech}[a+b x]\right]}{\sqrt{-1+a^2}}\right]+\right.\right.$$

$$\left.\left.\text{ArcTan}\left[\frac{\left(1+a\right) \text{Tanh}\left[\frac{1}{2} \text{ArcSech}[a+b x]\right]}{\sqrt{-1+a^2}}\right]\right)\right) \text{Log}\left[\frac{\sqrt{-1+a^2}\, e^{-\frac{1}{2} \text{ArcSech}[a+b x]}}{\sqrt{2} \sqrt{a} \sqrt{-\frac{b x}{a+b x}}}\right]+$$

$$\left(\text{ArcCos}\left[\frac{1}{a}\right]-2\left(\text{ArcTan}\left[\frac{(-1+a)\,\text{Coth}\left[\frac{1}{2}\text{ArcSech}[a+b x]\right]}{\sqrt{-1+a^2}}\right]+\text{ArcTan}\left[\frac{(1+a)\,\text{Tanh}\left[\frac{1}{2}\text{ArcSech}[a+b x]\right]}{\sqrt{-1+a^2}}\right]\right)\right)\text{Log}\left[\frac{\sqrt{-1+a^2}\;e^{\frac{1}{2}\text{ArcSech}[a+b x]}}{\sqrt{2}\,\sqrt{a}\,\sqrt{-\frac{b x}{a+b x}}}\right]-$$

$$\left(\text{ArcCos}\left[\frac{1}{a}\right]+2\,\text{ArcTan}\left[\frac{(1+a)\,\text{Tanh}\left[\frac{1}{2}\text{ArcSech}[a+b x]\right]}{\sqrt{-1+a^2}}\right]\right)$$

$$\text{Log}\left[-\left(\left((-1+a)\left(1+a-i\sqrt{-1+a^2}\right)\left(-1+\text{Tanh}\left[\frac{1}{2}\text{ArcSech}[a+b x]\right]\right)\right)\Big/\left(a\left(-1+a+i\sqrt{-1+a^2}\,\text{Tanh}\left[\frac{1}{2}\text{ArcSech}[a+b x]\right]\right)\right)\right)\right]-$$

$$\left(\text{ArcCos}\left[\frac{1}{a}\right]-2\,\text{ArcTan}\left[\frac{(1+a)\,\text{Tanh}\left[\frac{1}{2}\text{ArcSech}[a+b x]\right]}{\sqrt{-1+a^2}}\right]\right)$$

$$\text{Log}\left[\frac{(-1+a)\left(1+a+i\sqrt{-1+a^2}\right)\left(1+\text{Tanh}\left[\frac{1}{2}\text{ArcSech}[a+b x]\right]\right)}{a\left(-1+a+i\sqrt{-1+a^2}\,\text{Tanh}\left[\frac{1}{2}\text{ArcSech}[a+b x]\right]\right)}\right]+$$

$$i\left(\text{PolyLog}\left[2,\left(\left(-1-i\sqrt{-1+a^2}\right)\left(-1+a-i\sqrt{-1+a^2}\,\text{Tanh}\left[\frac{1}{2}\text{ArcSech}[a+b x]\right]\right)\right)\Big/\left(a\left(-1+a+i\sqrt{-1+a^2}\,\text{Tanh}\left[\frac{1}{2}\text{ArcSech}[a+b x]\right]\right)\right)\right]-\right.$$

$$\left.\text{PolyLog}\left[2,\left(\left(i+\sqrt{-1+a^2}\right)\left(-1+a-i\sqrt{-1+a^2}\,\text{Tanh}\left[\frac{1}{2}\text{ArcSech}[a+b x]\right]\right)\right)\Big/\left(a\left(-i(-1+a)+\sqrt{-1+a^2}\,\text{Tanh}\left[\frac{1}{2}\text{ArcSech}[a+b x]\right]\right)\right)\right]\right)\Bigg)+$$

$$\frac{1}{a^2\left(-1+a^2\right)^{3/2}}b^2\left(2\,\text{ArcSech}[a+b x]\,\text{ArcTan}\left[\frac{(-1+a)\,\text{Coth}\left[\frac{1}{2}\text{ArcSech}[a+b x]\right]}{\sqrt{-1+a^2}}\right]-\right.$$

$$2\,i\,\text{ArcCos}\left[\frac{1}{a}\right]\text{ArcTan}\left[\frac{(1+a)\,\text{Tanh}\left[\frac{1}{2}\text{ArcSech}[a+b x]\right]}{\sqrt{-1+a^2}}\right]+$$

$$\left(\text{ArcCos}\left[\frac{1}{a}\right]+2\left(\text{ArcTan}\left[\frac{(-1+a)\,\text{Coth}\left[\frac{1}{2}\text{ArcSech}[a+b x]\right]}{\sqrt{-1+a^2}}\right]+\right.\right.$$

$$\text{ArcTan}\left[\frac{(1+a)\,\text{Tanh}\left[\frac{1}{2}\text{ArcSech}[a+b\,x]\right]}{\sqrt{-1+a^2}}\right]\Bigg)\Bigg)\,\text{Log}\left[\frac{\sqrt{-1+a^2}\;e^{-\frac{1}{2}\text{ArcSech}[a+b\,x]}}{\sqrt{2}\,\sqrt{a}\,\sqrt{-\frac{b\,x}{a+b\,x}}}\right]+$$

$$\Bigg(\text{ArcCos}\left[\frac{1}{a}\right]-2\,\Bigg(\text{ArcTan}\left[\frac{(-1+a)\,\text{Coth}\left[\frac{1}{2}\text{ArcSech}[a+b\,x]\right]}{\sqrt{-1+a^2}}\right]+$$

$$\text{ArcTan}\left[\frac{(1+a)\,\text{Tanh}\left[\frac{1}{2}\text{ArcSech}[a+b\,x]\right]}{\sqrt{-1+a^2}}\right]\Bigg)\Bigg)\,\text{Log}\left[\frac{\sqrt{-1+a^2}\;e^{\frac{1}{2}\text{ArcSech}[a+b\,x]}}{\sqrt{2}\,\sqrt{a}\,\sqrt{-\frac{b\,x}{a+b\,x}}}\right]-$$

$$\left(\text{ArcCos}\left[\frac{1}{a}\right]+2\,\text{ArcTan}\left[\frac{(1+a)\,\text{Tanh}\left[\frac{1}{2}\text{ArcSech}[a+b\,x]\right]}{\sqrt{-1+a^2}}\right]\right)$$

$$\text{Log}\Bigg[-\Bigg(\Bigg((-1+a)\left(1+a-i\sqrt{-1+a^2}\right)\left(-1+\text{Tanh}\left[\frac{1}{2}\text{ArcSech}[a+b\,x]\right]\right)\Bigg)\Bigg/$$

$$\left(a\left(-1+a+i\sqrt{-1+a^2}\;\text{Tanh}\left[\frac{1}{2}\text{ArcSech}[a+b\,x]\right]\right)\right)\Bigg)\Bigg]-$$

$$\left(\text{ArcCos}\left[\frac{1}{a}\right]-2\,\text{ArcTan}\left[\frac{(1+a)\,\text{Tanh}\left[\frac{1}{2}\text{ArcSech}[a+b\,x]\right]}{\sqrt{-1+a^2}}\right]\right)$$

$$\text{Log}\left[\frac{(-1+a)\left(1+a+i\sqrt{-1+a^2}\right)\left(1+\text{Tanh}\left[\frac{1}{2}\text{ArcSech}[a+b\,x]\right]\right)}{a\left(-1+a+i\sqrt{-1+a^2}\;\text{Tanh}\left[\frac{1}{2}\text{ArcSech}[a+b\,x]\right]\right)}\right]+$$

$$i\,\Bigg(\text{PolyLog}\Bigg[2,\,\Bigg(\left(-1-i\sqrt{-1+a^2}\right)\left(-1+a-i\sqrt{-1+a^2}\;\text{Tanh}\left[\frac{1}{2}\text{ArcSech}[a+b\,x]\right]\right)\Bigg)\Bigg/$$

$$\left(a\left(-1+a+i\sqrt{-1+a^2}\;\text{Tanh}\left[\frac{1}{2}\text{ArcSech}[a+b\,x]\right]\right)\right)\Bigg]-$$

$$\text{PolyLog}\Bigg[2,\,\Bigg(\left(i+\sqrt{-1+a^2}\right)\left(-1+a-i\sqrt{-1+a^2}\;\text{Tanh}\left[\frac{1}{2}\text{ArcSech}[a+b\,x]\right]\right)\Bigg)\Bigg/$$

$$\left(a\left(-i\,(-1+a)+\sqrt{-1+a^2}\;\text{Tanh}\left[\frac{1}{2}\text{ArcSech}[a+b\,x]\right]\right)\right)\Bigg]\Bigg)\Bigg)$$

Problem 17: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int\frac{\text{ArcSech}[a+b\,x]^3}{x}\,dx$$

Optimal (type 4, 378 leaves, 20 steps):

$$\text{ArcSech}[a+b x]^3 \text{Log}\left[1-\frac{a\, e^{\text{ArcSech}[a+b x]}}{1-\sqrt{1-a^2}}\right]+\text{ArcSech}[a+b x]^3 \text{Log}\left[1-\frac{a\, e^{\text{ArcSech}[a+b x]}}{1+\sqrt{1-a^2}}\right]-$$
$$\text{ArcSech}[a+b x]^3 \text{Log}\left[1+e^{2 \text{ArcSech}[a+b x]}\right]+3 \text{ArcSech}[a+b x]^2 \text{PolyLog}\left[2,\frac{a\, e^{\text{ArcSech}[a+b x]}}{1-\sqrt{1-a^2}}\right]+$$
$$3 \text{ArcSech}[a+b x]^2 \text{PolyLog}\left[2,\frac{a\, e^{\text{ArcSech}[a+b x]}}{1+\sqrt{1-a^2}}\right]-\frac{3}{2} \text{ArcSech}[a+b x]^2 \text{PolyLog}\left[2,-e^{2 \text{ArcSech}[a+b x]}\right]-$$
$$6 \text{ArcSech}[a+b x] \text{PolyLog}\left[3,\frac{a\, e^{\text{ArcSech}[a+b x]}}{1-\sqrt{1-a^2}}\right]-6 \text{ArcSech}[a+b x] \text{PolyLog}\left[3,\frac{a\, e^{\text{ArcSech}[a+b x]}}{1+\sqrt{1-a^2}}\right]+$$
$$\frac{3}{2} \text{ArcSech}[a+b x] \text{PolyLog}\left[3,-e^{2 \text{ArcSech}[a+b x]}\right]+6 \text{PolyLog}\left[4,\frac{a\, e^{\text{ArcSech}[a+b x]}}{1-\sqrt{1-a^2}}\right]+$$
$$6 \text{PolyLog}\left[4,\frac{a\, e^{\text{ArcSech}[a+b x]}}{1+\sqrt{1-a^2}}\right]-\frac{3}{4} \text{PolyLog}\left[4,-e^{2 \text{ArcSech}[a+b x]}\right]$$

Result (type 4, 1025 leaves):

$$-\frac{1}{2} \text{ArcSech}[a+b x]^4-\text{ArcSech}[a+b x]^3 \text{Log}\left[1+e^{-2 \text{ArcSech}[a+b x]}\right]+$$
$$\text{ArcSech}[a+b x]^3 \text{Log}\left[1+\frac{\left(-1+\sqrt{1-a^2}\right) e^{-\text{ArcSech}[a+b x]}}{a}\right]+$$
$$6 i \text{ArcSech}[a+b x]^2 \text{ArcSin}\left[\frac{\sqrt{\frac{-1+a}{a}}}{\sqrt{2}}\right] \text{Log}\left[1+\frac{\left(-1+\sqrt{1-a^2}\right) e^{-\text{ArcSech}[a+b x]}}{a}\right]+$$
$$\text{ArcSech}[a+b x]^3 \text{Log}\left[1-\frac{\left(1+\sqrt{1-a^2}\right) e^{-\text{ArcSech}[a+b x]}}{a}\right]-$$
$$6 i \text{ArcSech}[a+b x]^2 \text{ArcSin}\left[\frac{\sqrt{\frac{-1+a}{a}}}{\sqrt{2}}\right] \text{Log}\left[1-\frac{\left(1+\sqrt{1-a^2}\right) e^{-\text{ArcSech}[a+b x]}}{a}\right]+$$
$$2 \text{ArcSech}[a+b x]^3 \text{Log}\left[1+\frac{a\, e^{\text{ArcSech}[a+b x]}}{-1+\sqrt{1-a^2}}\right]+2 \text{ArcSech}[a+b x]^3 \text{Log}\left[1-\frac{a\, e^{\text{ArcSech}[a+b x]}}{1+\sqrt{1-a^2}}\right]-$$
$$\text{ArcSech}[a+b x]^3 \text{Log}\left[1+\frac{\left(-1+\sqrt{1-a^2}\right)\left(1-\sqrt{-\frac{-1+a+b x}{1+a+b x}}\,(1+a+b x)\right)}{a\,(a+b x)}\right]-$$
$$6 i \text{ArcSech}[a+b x]^2 \text{ArcSin}\left[\frac{\sqrt{\frac{-1+a}{a}}}{\sqrt{2}}\right] \text{Log}\left[1+\frac{\left(-1+\sqrt{1-a^2}\right)\left(1-\sqrt{-\frac{-1+a+b x}{1+a+b x}}\,(1+a+b x)\right)}{a\,(a+b x)}\right]-$$
$$\text{ArcSech}[a+b x]^3 \text{Log}\left[1+\frac{\left(1+\sqrt{1-a^2}\right)\left(-1+\sqrt{-\frac{-1+a+b x}{1+a+b x}}\,(1+a+b x)\right)}{a\,(a+b x)}\right]+$$

$$6\,i\,\text{ArcSech}[a+b\,x]^2\,\text{ArcSin}\left[\frac{\sqrt{\frac{-1+a}{a}}}{\sqrt{2}}\right]\,\text{Log}\left[1+\frac{\left(1+\sqrt{1-a^2}\right)\left(-1+\sqrt{-\frac{-1+a+b\,x}{1+a+b\,x}}\,(1+a+b\,x)\right)}{a\,(a+b\,x)}\right]-$$

$$\text{ArcSech}[a+b\,x]^3\,\text{Log}\left[1+\frac{a\left(1+\sqrt{-\frac{-1+a+b\,x}{1+a+b\,x}}\,(1+a+b\,x)\right)}{\left(-1+\sqrt{1-a^2}\right)(a+b\,x)}\right]-$$

$$\text{ArcSech}[a+b\,x]^3\,\text{Log}\left[1-\frac{a\left(1+\sqrt{-\frac{-1+a+b\,x}{1+a+b\,x}}\,(1+a+b\,x)\right)}{\left(1+\sqrt{1-a^2}\right)(a+b\,x)}\right]+$$

$$\frac{3}{2}\,\text{ArcSech}[a+b\,x]^2\,\text{PolyLog}\left[2,-e^{-2\,\text{ArcSech}[a+b\,x]}\right]+$$

$$3\,\text{ArcSech}[a+b\,x]^2\,\text{PolyLog}\left[2,-\frac{a\,e^{\text{ArcSech}[a+b\,x]}}{-1+\sqrt{1-a^2}}\right]+3\,\text{ArcSech}[a+b\,x]^2\,\text{PolyLog}\left[2,\frac{a\,e^{\text{ArcSech}[a+b\,x]}}{1+\sqrt{1-a^2}}\right]+$$

$$\frac{3}{2}\,\text{ArcSech}[a+b\,x]\,\text{PolyLog}\left[3,-e^{-2\,\text{ArcSech}[a+b\,x]}\right]-6\,\text{ArcSech}[a+b\,x]\,\text{PolyLog}\left[3,-\frac{a\,e^{\text{ArcSech}[a+b\,x]}}{-1+\sqrt{1-a^2}}\right]-$$

$$6\,\text{ArcSech}[a+b\,x]\,\text{PolyLog}\left[3,\frac{a\,e^{\text{ArcSech}[a+b\,x]}}{1+\sqrt{1-a^2}}\right]+\frac{3}{4}\,\text{PolyLog}\left[4,-e^{-2\,\text{ArcSech}[a+b\,x]}\right]+$$

$$6\,\text{PolyLog}\left[4,-\frac{a\,e^{\text{ArcSech}[a+b\,x]}}{-1+\sqrt{1-a^2}}\right]+6\,\text{PolyLog}\left[4,\frac{a\,e^{\text{ArcSech}[a+b\,x]}}{1+\sqrt{1-a^2}}\right]$$

Problem 18: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int\frac{\text{ArcSech}[a+b\,x]^3}{x^2}\,dx$$

Optimal (type 4, 330 leaves, 14 steps):

$$-\frac{b\,\text{ArcSech}[a+b\,x]^3}{a}-\frac{\text{ArcSech}[a+b\,x]^3}{x}+$$

$$\frac{3\,b\,\text{ArcSech}[a+b\,x]^2\,\text{Log}\left[1-\frac{a\,e^{\text{ArcSech}[a+b\,x]}}{1-\sqrt{1-a^2}}\right]}{a\sqrt{1-a^2}}-\frac{3\,b\,\text{ArcSech}[a+b\,x]^2\,\text{Log}\left[1-\frac{a\,e^{\text{ArcSech}[a+b\,x]}}{1+\sqrt{1-a^2}}\right]}{a\sqrt{1-a^2}}+$$

$$\frac{6\,b\,\text{ArcSech}[a+b\,x]\,\text{PolyLog}\left[2,\frac{a\,e^{\text{ArcSech}[a+b\,x]}}{1-\sqrt{1-a^2}}\right]}{a\sqrt{1-a^2}}-\frac{6\,b\,\text{ArcSech}[a+b\,x]\,\text{PolyLog}\left[2,\frac{a\,e^{\text{ArcSech}[a+b\,x]}}{1+\sqrt{1-a^2}}\right]}{a\sqrt{1-a^2}}-$$

$$\frac{6\,b\,\text{PolyLog}\left[3,\frac{a\,e^{\text{ArcSech}[a+b\,x]}}{1-\sqrt{1-a^2}}\right]}{a\sqrt{1-a^2}}+\frac{6\,b\,\text{PolyLog}\left[3,\frac{a\,e^{\text{ArcSech}[a+b\,x]}}{1+\sqrt{1-a^2}}\right]}{a\sqrt{1-a^2}}$$

Result (type 4, 1779 leaves):

$$-\frac{1}{a\sqrt{-1+a^2}\,x}\Bigg(a\sqrt{-1+a^2}\,\text{ArcSech}[a+b\,x]^3+\sqrt{-1+a^2}\,b\,x\,\text{ArcSech}[a+b\,x]^3-6\,b\,x\,\text{ArcCos}\left[-\frac{1}{a}\right]$$

$$\text{ArcSech}[a+b\,x]\,\text{Log}\left[\frac{\sqrt{-1+a^2}\,e^{-i\,\text{ArcTan}\left[\frac{(1+a)\,\text{Tanh}\left[\frac{1}{2}\text{ArcSech}[a+b\,x]\right]}{\sqrt{-1+a^2}}\right]}}{\sqrt{2}\,\sqrt{a}\,\sqrt{1+a\,\text{Cos}\left[2\,\text{ArcTan}\left[\frac{(1+a)\,\text{Tanh}\left[\frac{1}{2}\text{ArcSech}[a+b\,x]\right]}{\sqrt{-1+a^2}}\right]\right]}}\right]+$$

$$12\,i\,b\,x\,\text{ArcSech}[a+b\,x]\,\text{ArcTanh}\left[\text{Coth}\left[\frac{1}{2}\text{ArcSech}[a+b\,x]\right]\right]$$

$$\text{Log}\left[\frac{\sqrt{-1+a^2}\,e^{-i\,\text{ArcTan}\left[\frac{(1+a)\,\text{Tanh}\left[\frac{1}{2}\text{ArcSech}[a+b\,x]\right]}{\sqrt{-1+a^2}}\right]}}{\sqrt{2}\,\sqrt{a}\,\sqrt{1+a\,\text{Cos}\left[2\,\text{ArcTan}\left[\frac{(1+a)\,\text{Tanh}\left[\frac{1}{2}\text{ArcSech}[a+b\,x]\right]}{\sqrt{-1+a^2}}\right]\right]}}\right]-$$

$$12\,i\,b\,x\,\text{ArcSech}[a+b\,x]\,\text{ArcTanh}\left[\text{Tanh}\left[\frac{1}{2}\text{ArcSech}[a+b\,x]\right]\right]$$

$$\text{Log}\left[\frac{\sqrt{-1+a^2}\,e^{-i\,\text{ArcTan}\left[\frac{(1+a)\,\text{Tanh}\left[\frac{1}{2}\text{ArcSech}[a+b\,x]\right]}{\sqrt{-1+a^2}}\right]}}{\sqrt{2}\,\sqrt{a}\,\sqrt{1+a\,\text{Cos}\left[2\,\text{ArcTan}\left[\frac{(1+a)\,\text{Tanh}\left[\frac{1}{2}\text{ArcSech}[a+b\,x]\right]}{\sqrt{-1+a^2}}\right]\right]}}\right]-6\,b\,x\,\text{ArcCos}\left[-\frac{1}{a}\right]$$

$$\text{ArcSech}[a+b\,x]\,\text{Log}\left[\frac{\sqrt{-1+a^2}\,e^{i\,\text{ArcTan}\left[\frac{(1+a)\,\text{Tanh}\left[\frac{1}{2}\text{ArcSech}[a+b\,x]\right]}{\sqrt{-1+a^2}}\right]}}{\sqrt{2}\,\sqrt{a}\,\sqrt{1+a\,\text{Cos}\left[2\,\text{ArcTan}\left[\frac{(1+a)\,\text{Tanh}\left[\frac{1}{2}\text{ArcSech}[a+b\,x]\right]}{\sqrt{-1+a^2}}\right]\right]}}\right]-$$

$$12\,i\,b\,x\,\text{ArcSech}[a+b\,x]\,\text{ArcTanh}\left[\text{Coth}\left[\frac{1}{2}\text{ArcSech}[a+b\,x]\right]\right]$$

$$\text{Log}\left[\frac{\sqrt{-1+a^2}\,e^{i\,\text{ArcTan}\left[\frac{(1+a)\,\text{Tanh}\left[\frac{1}{2}\text{ArcSech}[a+b\,x]\right]}{\sqrt{-1+a^2}}\right]}}{\sqrt{2}\,\sqrt{a}\,\sqrt{1+a\,\text{Cos}\left[2\,\text{ArcTan}\left[\frac{(1+a)\,\text{Tanh}\left[\frac{1}{2}\text{ArcSech}[a+b\,x]\right]}{\sqrt{-1+a^2}}\right]\right]}}\right]+$$

$$12\,i\,b\,x\,\text{ArcSech}[a+b\,x]\,\text{ArcTanh}\left[\text{Tanh}\left[\frac{1}{2}\text{ArcSech}[a+b\,x]\right]\right]$$

$$\text{Log}\left[\frac{\sqrt{-1+a^2}\; e^{i\,\text{ArcTan}\left[\frac{(1+a)\,\text{Tanh}\left[\frac{1}{2}\text{ArcSech}[a+b x]\right]}{\sqrt{-1+a^2}}\right]}}{\sqrt{2}\,\sqrt{a}\,\sqrt{1+a\,\text{Cos}\left[2\,\text{ArcTan}\left[\frac{(1+a)\,\text{Tanh}\left[\frac{1}{2}\text{ArcSech}[a+b x]\right]}{\sqrt{-1+a^2}}\right]\right]}}\right]+$$

$$6\,b\,x\,\text{ArcCos}\left[-\frac{1}{a}\right]\text{ArcSech}[a+b x]\,\text{Log}\left[\frac{\sqrt{-1+a^2}+i\,(1+a)\,\text{Tanh}\left[\frac{1}{2}\text{ArcSech}[a+b x]\right]}{2\sqrt{a}\,\sqrt{-\frac{(-1+a^2)\,(a+b x)}{b x}}\,\sqrt{-\frac{b x}{(-1+a)\,(1+a+b x)}}}\right]+$$

$$12\,i\,b\,x\,\text{ArcSech}[a+b x]\,\text{ArcTanh}\left[\text{Coth}\left[\frac{1}{2}\text{ArcSech}[a+b x]\right]\right]$$

$$\text{Log}\left[\frac{\sqrt{-1+a^2}+i\,(1+a)\,\text{Tanh}\left[\frac{1}{2}\text{ArcSech}[a+b x]\right]}{2\sqrt{a}\,\sqrt{-\frac{(-1+a^2)\,(a+b x)}{b x}}\,\sqrt{-\frac{b x}{(-1+a)\,(1+a+b x)}}}\right]-12\,i\,b\,x\,\text{ArcSech}[a+b x]$$

$$\text{ArcTanh}\left[\text{Tanh}\left[\frac{1}{2}\text{ArcSech}[a+b x]\right]\right]\text{Log}\left[\frac{\sqrt{-1+a^2}+i\,(1+a)\,\text{Tanh}\left[\frac{1}{2}\text{ArcSech}[a+b x]\right]}{2\sqrt{a}\,\sqrt{-\frac{(-1+a^2)\,(a+b x)}{b x}}\,\sqrt{-\frac{b x}{(-1+a)\,(1+a+b x)}}}\right]+$$

$$6\,b\,x\,\text{ArcCos}\left[-\frac{1}{a}\right]\text{ArcSech}[a+b x]\,\text{Log}\left[-\left(\left(i\,(-1+a^2)\sqrt{-\frac{b x}{(-1+a)\,(1+a+b x)}}\right)\right/\right.$$

$$\left.\left.\left(\sqrt{a}\,\sqrt{-\frac{(-1+a^2)\,(a+b x)}{b x}}\left(-i\sqrt{-1+a^2}+(1+a)\,\text{Tanh}\left[\frac{1}{2}\text{ArcSech}[a+b x]\right]\right)\right)\right)\right]-$$

$$12\,i\,b\,x\,\text{ArcSech}[a+b x]\,\text{ArcTanh}\left[\text{Coth}\left[\frac{1}{2}\text{ArcSech}[a+b x]\right]\right]$$

$$\text{Log}\left[-\left(\left(i\,(-1+a^2)\sqrt{-\frac{b x}{(-1+a)\,(1+a+b x)}}\right)\right/\right.$$

$$\left.\left.\left(\sqrt{a}\,\sqrt{-\frac{(-1+a^2)\,(a+b x)}{b x}}\left(-i\sqrt{-1+a^2}+(1+a)\,\text{Tanh}\left[\frac{1}{2}\text{ArcSech}[a+b x]\right]\right)\right)\right)\right]+$$

$$12\,i\,b\,x\,\text{ArcSech}[a+b x]\,\text{ArcTanh}\left[\text{Tanh}\left[\frac{1}{2}\text{ArcSech}[a+b x]\right]\right]$$

$$\text{Log}\left[-\left(\left(i\,(-1+a^2)\sqrt{-\frac{b x}{(-1+a)\,(1+a+b x)}}\right)\right/\right.$$

$$\left(\sqrt{a}\,\sqrt{-\frac{\left(-1+a^2\right)\,(a+b\,x)}{b\,x}}\,\left(-\mathrm{i}\,\sqrt{-1+a^2}+(1+a)\,\mathrm{Tanh}\left[\frac{1}{2}\,\mathrm{ArcSech}[a+b\,x]\right]\right)\right)\Bigg)\Bigg]-$$

$$3\,\mathrm{i}\,b\,x\,\mathrm{ArcSech}[a+b\,x]^2\,\mathrm{Log}\left[\frac{\left(-\mathrm{i}+\mathrm{i}\,a+\sqrt{-1+a^2}\right)\left(-\mathrm{i}+\frac{(1+a)\,\mathrm{Tanh}\left[\frac{1}{2}\mathrm{ArcSech}[a+b\,x]\right]}{\sqrt{-1+a^2}}\right)}{a\left(1+\mathrm{Tanh}\left[\frac{1}{2}\,\mathrm{ArcSech}[a+b\,x]\right]\right)}\right]+$$

$$3\,\mathrm{i}\,b\,x\,\mathrm{ArcSech}[a+b\,x]^2\,\mathrm{Log}\left[\frac{\left(\mathrm{i}-\mathrm{i}\,a+\sqrt{-1+a^2}\right)\left(\mathrm{i}+\frac{(1+a)\,\mathrm{Tanh}\left[\frac{1}{2}\mathrm{ArcSech}[a+b\,x]\right]}{\sqrt{-1+a^2}}\right)}{a\left(1+\mathrm{Tanh}\left[\frac{1}{2}\,\mathrm{ArcSech}[a+b\,x]\right]\right)}\right]-$$

$$6\,\mathrm{i}\,b\,x\,\mathrm{ArcSech}[a+b\,x]\,\mathrm{PolyLog}\left[2,\frac{\left(1-\mathrm{i}\,\sqrt{-1+a^2}\right)\left(1-\sqrt{-\frac{-1+a+b\,x}{1+a+b\,x}}\,(1+a+b\,x)\right)}{a\,(a+b\,x)}\right]+$$

$$6\,\mathrm{i}\,b\,x\,\mathrm{ArcSech}[a+b\,x]\,\mathrm{PolyLog}\left[2,\frac{\left(1+\mathrm{i}\,\sqrt{-1+a^2}\right)\left(1-\sqrt{-\frac{-1+a+b\,x}{1+a+b\,x}}\,(1+a+b\,x)\right)}{a\,(a+b\,x)}\right]-$$

$$6\,\mathrm{i}\,b\,x\,\mathrm{PolyLog}\left[3,\frac{\left(1-\mathrm{i}\,\sqrt{-1+a^2}\right)\left(1-\sqrt{-\frac{-1+a+b\,x}{1+a+b\,x}}\,(1+a+b\,x)\right)}{a\,(a+b\,x)}\right]+$$

$$6\,\mathrm{i}\,b\,x\,\mathrm{PolyLog}\left[3,\frac{\left(1+\mathrm{i}\,\sqrt{-1+a^2}\right)\left(1-\sqrt{-\frac{-1+a+b\,x}{1+a+b\,x}}\,(1+a+b\,x)\right)}{a\,(a+b\,x)}\right]\Bigg)$$

Problem 19: Unable to integrate problem.

$$\int \frac{\mathrm{ArcSech}[a+b\,x]^3}{x^3}\,dx$$

Optimal (type 4, 965 leaves, 32 steps):

$$-\frac{3 b^2 \text{ArcSech}[a+b x]^2}{2 a^2 \left(1-a^2\right)}+\frac{3 b^2 \sqrt{\frac{1-a-b x}{1+a+b x}} (1+a+b x) \text{ArcSech}[a+b x]^2}{2 a \left(1-a^2\right) (a+b x) \left(1-\frac{a}{a+b x}\right)}+$$

$$\frac{b^2 \text{ArcSech}[a+b x]^3}{2 a^2}-\frac{\text{ArcSech}[a+b x]^3}{2 x^2}+\frac{3 b^2 \text{ArcSech}[a+b x] \text{Log}\left[1-\frac{a e^{\text{ArcSech}[a+b x]}}{1-\sqrt{1-a^2}}\right]}{a^2 \left(1-a^2\right)}+$$

$$\frac{3 b^2 \text{ArcSech}[a+b x]^2 \text{Log}\left[1-\frac{a e^{\text{ArcSech}[a+b x]}}{1-\sqrt{1-a^2}}\right]}{2 a^2 \left(1-a^2\right)^{3/2}}-\frac{3 b^2 \text{ArcSech}[a+b x]^2 \text{Log}\left[1-\frac{a e^{\text{ArcSech}[a+b x]}}{1-\sqrt{1-a^2}}\right]}{a^2 \sqrt{1-a^2}}+$$

$$\frac{3 b^2 \text{ArcSech}[a+b x] \text{Log}\left[1-\frac{a e^{\text{ArcSech}[a+b x]}}{1+\sqrt{1-a^2}}\right]}{a^2 \left(1-a^2\right)}-\frac{3 b^2 \text{ArcSech}[a+b x]^2 \text{Log}\left[1-\frac{a e^{\text{ArcSech}[a+b x]}}{1+\sqrt{1-a^2}}\right]}{2 a^2 \left(1-a^2\right)^{3/2}}+$$

$$\frac{3 b^2 \text{ArcSech}[a+b x]^2 \text{Log}\left[1-\frac{a e^{\text{ArcSech}[a+b x]}}{1+\sqrt{1-a^2}}\right]}{a^2 \sqrt{1-a^2}}+\frac{3 b^2 \text{PolyLog}\left[2,\frac{a e^{\text{ArcSech}[a+b x]}}{1-\sqrt{1-a^2}}\right]}{a^2 \left(1-a^2\right)}+$$

$$\frac{3 b^2 \text{ArcSech}[a+b x] \text{PolyLog}\left[2,\frac{a e^{\text{ArcSech}[a+b x]}}{1-\sqrt{1-a^2}}\right]}{a^2 \left(1-a^2\right)^{3/2}}-\frac{6 b^2 \text{ArcSech}[a+b x] \text{PolyLog}\left[2,\frac{a e^{\text{ArcSech}[a+b x]}}{1-\sqrt{1-a^2}}\right]}{a^2 \sqrt{1-a^2}}+$$

$$\frac{3 b^2 \text{PolyLog}\left[2,\frac{a e^{\text{ArcSech}[a+b x]}}{1+\sqrt{1-a^2}}\right]}{a^2 \left(1-a^2\right)}-\frac{3 b^2 \text{ArcSech}[a+b x] \text{PolyLog}\left[2,\frac{a e^{\text{ArcSech}[a+b x]}}{1+\sqrt{1-a^2}}\right]}{a^2 \left(1-a^2\right)^{3/2}}+$$

$$\frac{6 b^2 \text{ArcSech}[a+b x] \text{PolyLog}\left[2,\frac{a e^{\text{ArcSech}[a+b x]}}{1+\sqrt{1-a^2}}\right]}{a^2 \sqrt{1-a^2}}-\frac{3 b^2 \text{PolyLog}\left[3,\frac{a e^{\text{ArcSech}[a+b x]}}{1-\sqrt{1-a^2}}\right]}{a^2 \left(1-a^2\right)^{3/2}}+$$

$$\frac{6 b^2 \text{PolyLog}\left[3,\frac{a e^{\text{ArcSech}[a+b x]}}{1-\sqrt{1-a^2}}\right]}{a^2 \sqrt{1-a^2}}+\frac{3 b^2 \text{PolyLog}\left[3,\frac{a e^{\text{ArcSech}[a+b x]}}{1+\sqrt{1-a^2}}\right]}{a^2 \left(1-a^2\right)^{3/2}}-\frac{6 b^2 \text{PolyLog}\left[3,\frac{a e^{\text{ArcSech}[a+b x]}}{1+\sqrt{1-a^2}}\right]}{a^2 \sqrt{1-a^2}}$$

Result (type 8, 14 leaves):

$$\int \frac{\text{ArcSech}[a+b x]^3}{x^3} \, dx$$

Problem 29: Result more than twice size of optimal antiderivative.

$$\int \frac{\text{ArcSech}\left[a x^n\right]}{x} \, dx$$

Optimal (type 4, 61 leaves, 7 steps):

$$\frac{\text{ArcSech}\left[a x^n\right]^2}{2 n}-\frac{\text{ArcSech}\left[a x^n\right] \text{Log}\left[1+e^{2 \text{ArcSech}\left[a x^n\right]}\right]}{n}-\frac{\text{PolyLog}\left[2,-e^{2 \text{ArcSech}\left[a x^n\right]}\right]}{2 n}$$

Result (type 4, 219 leaves):

$$\text{ArcSech}\left[a\,x^n\right]\,\text{Log}[x]+\frac{1}{8\left(n-a\,n\,x^n\right)}\sqrt{\frac{1-a\,x^n}{1+a\,x^n}}\left(4\sqrt{-1+a^2\,x^{2\,n}}\;\text{ArcTan}\left[\sqrt{-1+a^2\,x^{2\,n}}\right]\left(2\,n\,\text{Log}[x]-\text{Log}\left[a^2\,x^{2\,n}\right]\right)+\sqrt{1-a^2\,x^{2\,n}}\left(\text{Log}\left[a^2\,x^{2\,n}\right]^2-4\,\text{Log}\left[a^2\,x^{2\,n}\right]\,\text{Log}\left[\frac{1}{2}\left(1+\sqrt{1-a^2\,x^{2\,n}}\right)\right]+2\,\text{Log}\left[\frac{1}{2}\left(1+\sqrt{1-a^2\,x^{2\,n}}\right)\right]^2-4\,\text{PolyLog}\left[2,\frac{1}{2}-\frac{1}{2}\sqrt{1-a^2\,x^{2\,n}}\right]\right)\right)$$

Problem 31: Result more than twice size of optimal antiderivative.

$$\int \text{ArcSech}\left[c\,e^{a+b\,x}\right]\,dx$$

Optimal (type 4, 77 leaves, 7 steps):

$$\frac{\text{ArcSech}\left[c\,e^{a+b\,x}\right]^2}{2\,b}-\frac{\text{ArcSech}\left[c\,e^{a+b\,x}\right]\,\text{Log}\left[1+e^{2\,\text{ArcSech}\left[c\,e^{a+b\,x}\right]}\right]}{b}-\frac{\text{PolyLog}\left[2,-e^{2\,\text{ArcSech}\left[c\,e^{a+b\,x}\right]}\right]}{2\,b}$$

Result (type 4, 249 leaves):

$$x\,\text{ArcSech}\left[c\,e^{a+b\,x}\right]-\frac{1}{8\,b\sqrt{1-c\,e^{a+b\,x}}}\sqrt{\frac{1-c\,e^{a+b\,x}}{1+c\,e^{a+b\,x}}}\sqrt{1+c\,e^{a+b\,x}}\left(\text{ArcTanh}\left[\sqrt{1-c^2\,e^{2\,(a+b\,x)}}\right]\left(8\,b\,x-4\,\text{Log}\left[c^2\,e^{2\,(a+b\,x)}\right]\right)-\text{Log}\left[c^2\,e^{2\,(a+b\,x)}\right]^2+4\,\text{Log}\left[c^2\,e^{2\,(a+b\,x)}\right]\,\text{Log}\left[\frac{1}{2}\left(1+\sqrt{1-c^2\,e^{2\,(a+b\,x)}}\right)\right]-2\,\text{Log}\left[\frac{1}{2}\left(1+\sqrt{1-c^2\,e^{2\,(a+b\,x)}}\right)\right]^2+4\,\text{PolyLog}\left[2,\frac{1}{2}\left(1-\sqrt{1-c^2\,e^{2\,(a+b\,x)}}\right)\right]\right)$$

Problem 33: Result unnecessarily involves imaginary or complex numbers.

$$\int e^{\text{ArcSech}[a\,x]}\,x^3\,dx$$

Optimal (type 3, 84 leaves, 5 steps):

$$\frac{x^3}{12\,a}+\frac{1}{4}\,e^{\text{ArcSech}[a\,x]}\,x^4-\frac{x\sqrt{1-a\,x}}{8\,a^3\sqrt{\frac{1}{1+a\,x}}}+\frac{\sqrt{\frac{1}{1+a\,x}}\,\sqrt{1+a\,x}\;\text{ArcSin}[a\,x]}{8\,a^4}$$

Result (type 3, 97 leaves):

$$\frac{1}{24 a^4}\left(8 a^3 x^3-3 a \sqrt{\frac{1-a x}{1+a x}} \left(x+a x^2-2 a^2 x^3-2 a^3 x^4\right)+3 i \text{Log}\left[-2 i a x+2 \sqrt{\frac{1-a x}{1+a x}} (1+a x)\right]\right)$$

Problem 35: Result unnecessarily involves imaginary or complex numbers.

$$\int e^{\text{ArcSech}[a x]} x \, dx$$

Optimal (type 3, 53 leaves, 4 steps):

$$\frac{x}{2 a}+\frac{1}{2} e^{\text{ArcSech}[a x]} x^2+\frac{\sqrt{\frac{1}{1+a x}} \sqrt{1+a x} \text{ArcSin}[a x]}{2 a^2}$$

Result (type 3, 75 leaves):

$$\frac{2 a x+a x \sqrt{\frac{1-a x}{1+a x}} (1+a x)+i \text{Log}\left[-2 i a x+2 \sqrt{\frac{1-a x}{1+a x}} (1+a x)\right]}{2 a^2}$$

Problem 36: Result more than twice size of optimal antiderivative.

$$\int e^{\text{ArcSech}[a x]} \, dx$$

Optimal (type 3, 24 leaves, 3 steps):

$$e^{\text{ArcSech}[a x]} x-\frac{\text{ArcSech}[a x]}{a}+\frac{\text{Log}[x]}{a}$$

Result (type 3, 79 leaves):

$$\frac{\sqrt{\frac{1-a x}{1+a x}} (1+a x)+2 \text{Log}[a x]-\text{Log}\left[1+\sqrt{\frac{1-a x}{1+a x}}+a x \sqrt{\frac{1-a x}{1+a x}}\right]}{a}$$

Problem 37: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{e^{\text{ArcSech}[a x]}}{x} \, dx$$

Optimal (type 3, 48 leaves, 5 steps):

$$-\frac{2}{1-\sqrt{\frac{1-a x}{1+a x}}}+2 \text{ArcTan}\left[\sqrt{\frac{1-a x}{1+a x}}\right]$$

Result (type 3, 75 leaves):

$$-\frac{1}{a\,x}+\left(-1-\frac{1}{a\,x}\right)\sqrt{\frac{1-a\,x}{1+a\,x}}-\mathrm{i}\,\mathrm{Log}\left[-2\,\mathrm{i}\,a\,x+2\sqrt{\frac{1-a\,x}{1+a\,x}}\,(1+a\,x)\right]$$

Problem 38: Result more than twice size of optimal antiderivative.

$$\int\frac{e^{\mathrm{ArcSech}[a\,x]}}{x^2}\,dx$$

Optimal (type 3, 35 leaves, 6 steps):

$$-\frac{e^{\mathrm{ArcSech}[a\,x]}}{2\,x}+a\,\mathrm{ArcTanh}\left[\sqrt{\frac{1-a\,x}{1+a\,x}}\right]$$

Result (type 3, 93 leaves):

$$\frac{1}{2}\left(-\frac{1}{a\,x^2}-\frac{\sqrt{\frac{1-a\,x}{1+a\,x}}\,(1+a\,x)}{a\,x^2}-a\,\mathrm{Log}[x]+a\,\mathrm{Log}\left[1+\sqrt{\frac{1-a\,x}{1+a\,x}}+a\,x\sqrt{\frac{1-a\,x}{1+a\,x}}\right]\right)$$

Problem 45: Result unnecessarily involves imaginary or complex numbers.

$$\int e^{\mathrm{ArcSech}[a\,x^2]}\,x^7\,dx$$

Optimal (type 3, 111 leaves, 6 steps):

$$\frac{x^6}{24\,a}+\frac{1}{8}\,e^{\mathrm{ArcSech}[a\,x^2]}\,x^8-\frac{x^2\sqrt{\frac{1}{1+a\,x^2}}\,\sqrt{1+a\,x^2}\,\sqrt{1-a^2\,x^4}}{16\,a^3}+\frac{\sqrt{\frac{1}{1+a\,x^2}}\,\sqrt{1+a\,x^2}\,\mathrm{ArcSin}[a\,x^2]}{16\,a^4}$$

Result (type 3, 111 leaves):

$$\frac{1}{48\,a^4}\left(8\,a^3\,x^6-3\,a\sqrt{\frac{1-a\,x^2}{1+a\,x^2}}\,\left(x^2+a\,x^4-2\,a^2\,x^6-2\,a^3\,x^8\right)+3\,\mathrm{i}\,\mathrm{Log}\left[-2\,\mathrm{i}\,a\,x^2+2\sqrt{\frac{1-a\,x^2}{1+a\,x^2}}\,\left(1+a\,x^2\right)\right]\right)$$

Problem 46: Result unnecessarily involves imaginary or complex numbers.

$$\int e^{\mathrm{ArcSech}[a\,x^2]}\,x^6\,dx$$

Optimal (type 4, 115 leaves, 5 steps):

$$\frac{2x^5}{35a}+\frac{1}{7}e^{\text{ArcSech}[a x^2]}x^7-\frac{2x\sqrt{\frac{1}{1+a x^2}}\sqrt{1+a x^2}\sqrt{1-a^2x^4}}{21a^3}+\frac{2\sqrt{\frac{1}{1+a x^2}}\sqrt{1+a x^2}\,\text{EllipticF}[\text{ArcSin}[\sqrt{a}\,x],-1]}{21a^{7/2}}$$

Result (type 4, 139 leaves):

$$\frac{x^5}{5a}+\frac{x\sqrt{\frac{1-a x^2}{1+a x^2}}\left(-2-2a x^2+3a^2x^4+3a^3x^6\right)}{21a^3}-\frac{2\,i\sqrt{\frac{1-a x^2}{1+a x^2}}\sqrt{1-a^2x^4}\,\text{EllipticF}[i\,\text{ArcSinh}[\sqrt{-a}\,x],-1]}{21(-a)^{7/2}\left(-1+a x^2\right)}$$

Problem 48: Result unnecessarily involves imaginary or complex numbers.

$$\int e^{\text{ArcSech}[a x^2]}x^4\,dx$$

Optimal (type 4, 112 leaves, 7 steps):

$$\frac{2x^3}{15a}+\frac{1}{5}e^{\text{ArcSech}[a x^2]}x^5+\frac{2\sqrt{\frac{1}{1+a x^2}}\sqrt{1+a x^2}\,\text{EllipticE}[\text{ArcSin}[\sqrt{a}\,x],-1]}{5a^{5/2}}-\frac{2\sqrt{\frac{1}{1+a x^2}}\sqrt{1+a x^2}\,\text{EllipticF}[\text{ArcSin}[\sqrt{a}\,x],-1]}{5a^{5/2}}$$

Result (type 4, 140 leaves):

$$\frac{1}{15}\left(\frac{5x^3}{a}+\frac{3\sqrt{\frac{1-a x^2}{1+a x^2}}\left(x^3+a x^5\right)}{a}+\left(6\,i\sqrt{\frac{1-a x^2}{1+a x^2}}\sqrt{1-a^2x^4}\left(\text{EllipticE}[i\,\text{ArcSinh}[\sqrt{-a}\,x],-1]-\text{EllipticF}[i\,\text{ArcSinh}[\sqrt{-a}\,x],-1]\right)\right)\Big/\left((-a)^{5/2}\left(-1+a x^2\right)\right)\right)$$

Problem 49: Result unnecessarily involves imaginary or complex numbers.

$$\int e^{\text{ArcSech}[a x^2]} x^3 \, dx$$

Optimal (type 3, 63 leaves, 5 steps):

$$\frac{x^2}{4 a}+\frac{1}{4} e^{\text{ArcSech}[a x^2]} x^4+\frac{\sqrt{\frac{1}{1+a x^2}} \sqrt{1+a x^2} \text{ ArcSin}[a x^2]}{4 a^2}$$

Result (type 3, 92 leaves):

$$\frac{2 a x^2+a \sqrt{\frac{1-a x^2}{1+a x^2}} \left(x^2+a x^4\right)+i \text{ Log}\left[-2 i a x^2+2 \sqrt{\frac{1-a x^2}{1+a x^2}} \left(1+a x^2\right)\right]}{4 a^2}$$

Problem 50: Result unnecessarily involves imaginary or complex numbers.

$$\int e^{\text{ArcSech}[a x^2]} x^2 \, dx$$

Optimal (type 4, 67 leaves, 4 steps):

$$\frac{2 x}{3 a}+\frac{1}{3} e^{\text{ArcSech}[a x^2]} x^3+\frac{2 \sqrt{\frac{1}{1+a x^2}} \sqrt{1+a x^2} \text{ EllipticF}\left[\text{ArcSin}\left[\sqrt{a} x\right],-1\right]}{3 a^{3/2}}$$

Result (type 4, 116 leaves):

$$\frac{x}{a}+\frac{\sqrt{\frac{1-a x^2}{1+a x^2}} \left(x+a x^3\right)}{3 a}-\frac{2 i \sqrt{\frac{1-a x^2}{1+a x^2}} \sqrt{1-a^2 x^4} \text{ EllipticF}\left[i \text{ ArcSinh}\left[\sqrt{-a} x\right],-1\right]}{3 (-a)^{3/2} \left(-1+a x^2\right)}$$

Problem 52: Result unnecessarily involves imaginary or complex numbers.

$$\int e^{\text{ArcSech}[a x^2]} \, dx$$

Optimal (type 4, 147 leaves, 8 steps):

$$-\frac{2}{a x}+e^{\text{ArcSech}[a x^2]} x-\frac{2 \sqrt{\frac{1}{1+a x^2}} \sqrt{1+a x^2} \sqrt{1-a^2 x^4}}{a x}-\frac{2 \sqrt{\frac{1}{1+a x^2}} \sqrt{1+a x^2} \text{EllipticE}\left[\text{ArcSin}\left[\sqrt{a} x\right],-1\right]}{\sqrt{a}}+\frac{2 \sqrt{\frac{1}{1+a x^2}} \sqrt{1+a x^2} \text{EllipticF}\left[\text{ArcSin}\left[\sqrt{a} x\right],-1\right]}{\sqrt{a}}$$

Result (type 4, 135 leaves):

$$-\frac{1}{a x}+\left(-\frac{1}{a x}-x\right) \sqrt{\frac{1-a x^2}{1+a x^2}}-\frac{1}{\sqrt{-a} \left(-1+a x^2\right)} 2 i \sqrt{\frac{1-a x^2}{1+a x^2}} \sqrt{1-a^2 x^4} \left(\text{EllipticE}\left[i \text{ArcSinh}\left[\sqrt{-a} x\right],-1\right]-\text{EllipticF}\left[i \text{ArcSinh}\left[\sqrt{-a} x\right],-1\right]\right)$$

Problem 54: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{e^{\text{ArcSech}[a x^2]}}{x^2} \, dx$$

Optimal (type 4, 115 leaves, 5 steps):

$$\frac{2}{3 a x^3}-\frac{e^{\text{ArcSech}[a x^2]}}{x}+\frac{2 \sqrt{\frac{1}{1+a x^2}} \sqrt{1+a x^2} \sqrt{1-a^2 x^4}}{3 a x^3}-\frac{2}{3} \sqrt{a} \sqrt{\frac{1}{1+a x^2}} \sqrt{1+a x^2} \text{EllipticF}\left[\text{ArcSin}\left[\sqrt{a} x\right],-1\right]$$

Result (type 4, 123 leaves):

$$-\frac{1}{3 a x^3}-\frac{\sqrt{\frac{1-a x^2}{1+a x^2}} \left(1+a x^2\right)}{3 a x^3}+\frac{2 i \sqrt{-a} \sqrt{\frac{1-a x^2}{1+a x^2}} \sqrt{1-a^2 x^4} \text{EllipticF}\left[i \text{ArcSinh}\left[\sqrt{-a} x\right],-1\right]}{-3+3 a x^2}$$

Problem 58: Unable to integrate problem.

$$\int e^{\text{ArcSech}[a x]} x^m \, dx$$

Optimal (type 5, 91 leaves, 4 steps):

$$\frac{x^m}{a\,m\,(1+m)}+\frac{e^{\text{ArcSech}[a\,x]}\,x^{1+m}}{1+m}+\frac{x^m\sqrt{\frac{1}{1+a\,x}}\,\sqrt{1+a\,x}\;\text{Hypergeometric2F1}\left[\frac{1}{2},\frac{m}{2},\frac{2+m}{2},a^2\,x^2\right]}{a\,m\,(1+m)}$$

Result (type 8, 12 leaves):

$$\int e^{\text{ArcSech}[a\,x]}\,x^m\,dx$$

Problem 63: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{e^{\text{ArcSech}[a\,x^p]}}{x}\,dx$$

Optimal (type 3, 87 leaves, 6 steps):

$$-\frac{x^{-p}}{a\,p}-\frac{x^{-p}\sqrt{1-a\,x^p}}{a\,p\sqrt{\frac{1}{1+a\,x^p}}}-\frac{\sqrt{\frac{1}{1+a\,x^p}}\,\sqrt{1+a\,x^p}\;\text{ArcSin}[a\,x^p]}{p}$$

Result (type 3, 96 leaves):

$$-\frac{1}{a\,p}i\left(-i\,x^{-p}-i\,(a+x^{-p})\sqrt{\frac{1-a\,x^p}{1+a\,x^p}}+a\,\text{Log}\left[-2\,i\,a\,x^p+2\sqrt{\frac{1-a\,x^p}{1+a\,x^p}}\,(1+a\,x^p)\right]\right)$$

Problem 65: Result unnecessarily involves imaginary or complex numbers.

$$\int e^{2\,\text{ArcSech}[a\,x]}\,x^4\,dx$$

Optimal (type 3, 203 leaves, 9 steps):

$$\frac{5\sqrt{\frac{1-a\,x}{1+a\,x}}\,(1+a\,x)^2}{4\,a^5}+\frac{(1-a\,x)\,(1+a\,x)^4}{5\,a^5}+\frac{\sqrt{\frac{1-a\,x}{1+a\,x}}\,(1+a\,x)^4\left(5-6\sqrt{\frac{1-a\,x}{1+a\,x}}\right)}{10\,a^5}+$$

$$\frac{(1+a\,x)\left(4-\sqrt{\frac{1-a\,x}{1+a\,x}}\right)}{4\,a^5}-\frac{(1+a\,x)^3\left(4+45\sqrt{\frac{1-a\,x}{1+a\,x}}\right)}{30\,a^5}-\frac{\text{ArcTan}\left[\sqrt{\frac{1-a\,x}{1+a\,x}}\right]}{2\,a^5}$$

Result (type 3, 105 leaves):

$$\frac{1}{60\,a^5}\left(40\,a^3\,x^3-12\,a^5\,x^5-\right.$$

$$\left.15\,a\sqrt{\frac{1-a\,x}{1+a\,x}}\,(x+a\,x^2-2\,a^2\,x^3-2\,a^3\,x^4)+15\,i\,\text{Log}\left[-2\,i\,a\,x+2\sqrt{\frac{1-a\,x}{1+a\,x}}\,(1+a\,x)\right]\right)$$

Problem 67: Result unnecessarily involves imaginary or complex numbers.

$$\int e^{2\,\text{ArcSech}[a\,x]}\,x^2\,dx$$

Optimal (type 3, 169 leaves, 7 steps):

$$\frac{(1+a\,x)\left(1-\sqrt{\frac{1-a\,x}{1+a\,x}}\right)\left(1+\sqrt{\frac{1-a\,x}{1+a\,x}}\right)}{2\,a^3}-\frac{\sqrt{\frac{1-a\,x}{1+a\,x}}\,(1+a\,x)^2\left(1+\sqrt{\frac{1-a\,x}{1+a\,x}}\right)^3}{6\,a^3}+\frac{(1+a\,x)^3\left(1+\sqrt{\frac{1-a\,x}{1+a\,x}}\right)^4}{12\,a^3}-\frac{2\,\text{ArcTan}\left[\sqrt{\frac{1-a\,x}{1+a\,x}}\right]}{a^3}$$

Result (type 3, 86 leaves):

$$\frac{2\,x}{a^2}-\frac{x^3}{3}+\sqrt{\frac{1-a\,x}{1+a\,x}}\left(\frac{x}{a^2}+\frac{x^2}{a}\right)+\frac{i\,\text{Log}\left[-2\,i\,a\,x+2\sqrt{\frac{1-a\,x}{1+a\,x}}\,(1+a\,x)\right]}{a^3}$$

Problem 77: Result unnecessarily involves imaginary or complex numbers.

$$\int e^{-\text{ArcSech}[a\,x]}\,x^3\,dx$$

Optimal (type 3, 163 leaves, 7 steps):

$$-\frac{\sqrt{\frac{1-a\,x}{1+a\,x}}\,(1+a\,x)^4}{4\,a^4}+\frac{(1+a\,x)\left(8+\sqrt{\frac{1-a\,x}{1+a\,x}}\right)}{8\,a^4}-\frac{(1+a\,x)^2\left(8+5\sqrt{\frac{1-a\,x}{1+a\,x}}\right)}{8\,a^4}+\frac{(1+a\,x)^3\left(4+9\sqrt{\frac{1-a\,x}{1+a\,x}}\right)}{12\,a^4}+\frac{\text{ArcTan}\left[\sqrt{\frac{1-a\,x}{1+a\,x}}\right]}{4\,a^4}$$

Result (type 3, 97 leaves):

$$\frac{1}{24\,a^4}\left(8\,a^3\,x^3+3\,a\sqrt{\frac{1-a\,x}{1+a\,x}}\,\left(x+a\,x^2-2\,a^2\,x^3-2\,a^3\,x^4\right)-3\,i\,\text{Log}\left[-2\,i\,a\,x+2\sqrt{\frac{1-a\,x}{1+a\,x}}\,(1+a\,x)\right]\right)$$

Problem 79: Result unnecessarily involves imaginary or complex numbers.

$$\int e^{-\text{ArcSech}[a\,x]}\,x\,dx$$

Optimal (type 3, 94 leaves, 5 steps):

$$\frac{(1+a x)^2\left(1-\sqrt{\frac{1-a x}{1+a x}}\right)^2}{4 a^2}+\frac{(1+a x)\left(1+\sqrt{\frac{1-a x}{1+a x}}\right)}{2 a^2}+\frac{\text{ArcTan}\left[\sqrt{\frac{1-a x}{1+a x}}\right]}{a^2}$$

Result (type 3, 75 leaves):

$$-\frac{-2 a x+a x\sqrt{\frac{1-a x}{1+a x}}(1+a x)+i\,\text{Log}\left[-2 i a x+2\sqrt{\frac{1-a x}{1+a x}}(1+a x)\right]}{2 a^2}$$

Problem 81: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{e^{-\text{ArcSech}[a x]}}{x}\,dx$$

Optimal (type 3, 46 leaves, 5 steps):

$$-\frac{2}{1+\sqrt{\frac{1-a x}{1+a x}}}-2\,\text{ArcTan}\left[\sqrt{\frac{1-a x}{1+a x}}\right]$$

Result (type 3, 74 leaves):

$$-\frac{1}{a x}+\left(1+\frac{1}{a x}\right)\sqrt{\frac{1-a x}{1+a x}}+i\,\text{Log}\left[-2 i a x+2\sqrt{\frac{1-a x}{1+a x}}(1+a x)\right]$$

Problem 88: Unable to integrate problem.

$$\int\frac{e^{\text{ArcSech}[c x]}(d x)^m}{1-c^2 x^2}\,dx$$

Optimal (type 5, 89 leaves, 5 steps):

$$\frac{(d x)^m\sqrt{\frac{1}{1+c x}}\sqrt{1+c x}\,\text{Hypergeometric2F1}\left[\frac{1}{2},\frac{m}{2},\frac{2+m}{2},c^2 x^2\right]}{c m}+\frac{(d x)^m\,\text{Hypergeometric2F1}\left[1,\frac{m}{2},\frac{2+m}{2},c^2 x^2\right]}{c m}$$

Result (type 8, 26 leaves):

$$\int\frac{e^{\text{ArcSech}[c x]}(d x)^m}{1-c^2 x^2}\,dx$$

Problem 90: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{e^{\text{ArcSech}[c x]}x^3}{1-c^2 x^2}\,dx$$

Optimal (type 3, 75 leaves, 7 steps):

$$-\frac{x}{c^3}-\frac{x\sqrt{1-c\,x}}{2\,c^3\sqrt{\frac{1}{1+c\,x}}}+\frac{\sqrt{\frac{1}{1+c\,x}}\ \sqrt{1+c\,x}\ \text{ArcSin}[c\,x]}{2\,c^4}+\frac{\text{ArcTanh}[c\,x]}{c^4}$$

Result (type 3, 110 leaves):

$$-\frac{1}{2\,c^4}\left(2\,c\,x+c\,x\sqrt{\frac{1-c\,x}{1+c\,x}}+c^2\,x^2\sqrt{\frac{1-c\,x}{1+c\,x}}+\text{Log}[1-c\,x]-\text{Log}[1+c\,x]-\mathbb{i}\,\text{Log}\left[-2\,\mathbb{i}\,c\,x+2\sqrt{\frac{1-c\,x}{1+c\,x}}\ (1+c\,x)\right]\right)$$

Problem 92: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{e^{\text{ArcSech}[c\,x]}\,x}{1-c^2\,x^2}\,dx$$

Optimal (type 3, 37 leaves, 5 steps):

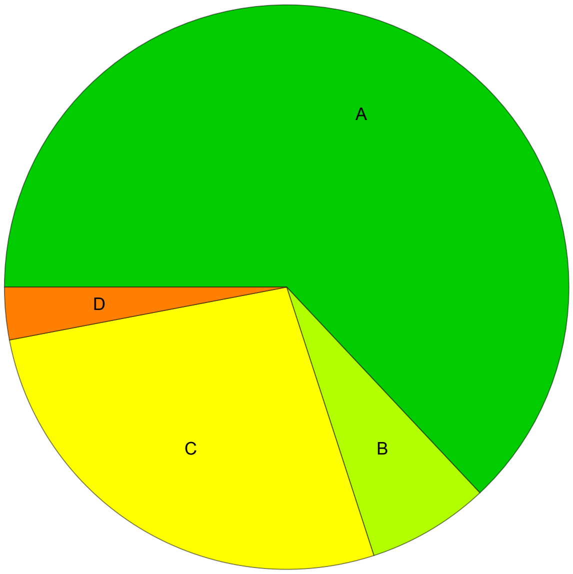
$$\frac{\sqrt{\frac{1}{1+c\,x}}\ \sqrt{1+c\,x}\ \text{ArcSin}[c\,x]}{c^2}+\frac{\text{ArcTanh}[c\,x]}{c^2}$$

Result (type 3, 68 leaves):

$$-\frac{\text{Log}[1-c\,x]}{2\,c^2}+\frac{\text{Log}[1+c\,x]}{2\,c^2}+\frac{\mathbb{i}\,\text{Log}\left[-2\,\mathbb{i}\,c\,x+2\sqrt{\frac{1-c\,x}{1+c\,x}}\ (1+c\,x)\right]}{c^2}$$

Summary of Integration Test Results

100 integration problems



A - 63 optimal antiderivatives

B - 7 more than twice size of optimal antiderivatives

C - 27 unnecessarily complex antiderivatives

D - 3 unable to integrate problems

E - 0 integration timeouts